Letters

Comments on "Calculation of Cutoff Wavenumbers for TE and TM Modes in Tubular Lines with Offset Center Conductor"

J. G. FIKIORIS AND J.A. ROUMELIOTIS

In the paper[1] by Vishen *et al.* [1], the authors refer to two papers of ours, numbered [8] and [10] in their list of references, and apparently without any elaboration make certain unwarranted comments on the method and results of [8], namely: "Some of the tabulated parameters of [8] actually yield nonphysical results; also, symmetric and antisymmetric modes appear to be degenerate, which contradicts experimental observations."

Before refuting these remarks, let us point out certain errors and misstatements of the article. In [1, eq. (1)], the authors define k as the wavenumber of the medium. It is actually $k_c = (\omega^2 \mu\epsilon - \beta^2)^{1/2}$. Otherwise, above cutoff the transverse $(r, \theta$ or $x, y)$ dependence of the field of the mode becomes a function of frequency, whereas it is solely, through k_c, a function of the cross-sectional geometry of the guide. This is not a simple typographical error, because the authors explicitly define k as $\omega\sqrt{\mu\epsilon}$, the wavenumber.

A misstatement also appears in the last paragraph of their introduction. They claim a rigorous mathematical derivation starting from the Helmholtz equation (with k instead of k_c!). Their method, however, in no way differs from the approach followed by many authors: separation of variables and use of the well-known addition theorems for Bessel functions to satisfy the boundary condition at the second circular boundary. The infinite set of linear equations obtained in this way is the same in all papers, including ours; no one can claim originality at this point.

They proceed by solving the set numerically, a procedure requiring repetition each time the eccentricity d changes. In [8] we chose to solve it analytically for *small* eccentricities $k_c d$, preserving a generality—inherent in all analytical methods—that dispenses with the need to solve numerically a large set of equations each time d changes. How restricted our results are because of this limitation is discussed in detail in [8] and is shown to provide very good results for appreciable $k_c d$. This is further corroborated by the results of Vishen *et al.*, which are in very good agreement with our results from [8] as long as $k_c d$ is not too large. Table I here compares results from Table I (TE modes) in Vishen *et al.*, Kuttler's lower and upper bounds, and our own from [8]. The agreement for $k_c d < 1$ (sometimes even for $k_c d > 1$) is remarkable. In certain cases, our results are within Kuttler's bounds, whereas those of Vishen *et al.* (marked by an asterisk) fall outside. Also, missing roots in Vishen *et al.* (marked by a bar) are supplied by our method; so, in the third case ($n = 0.5$, $d = 0.2$) our result is $k_{c41} = 5.1059$ and falls very near Kuttler's bounds. Vishen *et al.* miss this zero. In general, they fail to provide any kind of explanation for these two exceptional cases (asterisks and bars.) It is also obvious that our results start to deviate only when $k_c d$ exceeds 1. This is, also, the prevailing case for the values of Table II (TM modes) in [8] and no meaningful comparison was possible. Even there, the tendency to obtain similar results was obvious as $k_c d$ became smaller.

TABLE I
CUTOFF WAVENUMBERS FOR TE MODES

		Symmetric Modes					Antisymmetric Modes				
	Kcmn	Vishen et al	Kuttler's bounds Lower	Upper	Ours from [8] Kcmn	Kcd	Vishen et al	Kuttler's bounds Lower	Upper	Ours from [8] Kcmn	Kcd
$n = \frac{2}{3}$ $d = 0.2$	11	1.2522*	1 32027	1.32221	1.30227	0 242	1 1917	1 19001	1 19176	1.19524	0 242
	21	2 4365	2 4408	2 4446	2 43136	0 482	2 4307*	2 4267	2 4305	2 43136	0 482
	31	3 6209	3 6157	3 6218	3 6180	0 722	3.6203	3 6142	3 6203	3 6180	0 722
	41	4 7897	4.7804	4 7901	4 78866	0 960	4.7896	4.7804	4 7899	4.78866	0 960
	51	5 9379	5 9218	5 9385	5 9370	1 195	5 9379	5 9231	5.9385-	5 9370	1 195
$n = \frac{1}{3}$ $d = \frac{2}{9}$	11	1 5619*	1 5766	1 5807	1 57903	0 342	1 5435	1 5393	1 5436	1 54373	0 342
	21	2 9064	2 8968	2 9067	2 90555	0 651	2 9058	2.8966	2 9067	2 90555	0 651
	31	4 1152	4 0944	4 1161	4 10792	0 925	4 1152	4 0955	4 1161	4 10792	0 925
	41	4 4220*	4 2146	4 2356	5 26892	1 179	5.1606	5 131	5 167	5 26892	1 179
	51	5.2669	5.219	5.270	6 3957	1 425	5 2758	5.237	5 279	6 3957	1.425
$n = 0.5$ $d = 0\ 2$	11	1.3793*	1.40694	1 40793	1.40408	0 271	1.3522	1.35114	1.35219	1 35278	0 271
	21	2 6849	2 6837	2 6862	2.6844	0 536	2 6838	2 6815	2 6840	2 6844	0 536
	31	3 9295	3 9247	3 9298	3.9288	0 792	3.9296	3 9247	3.9298	3 9288	0.792
	41	-	4 9937	5 0192	5 1059	1 035	5 1131	5.1036	5 1138	5 1059	1 035
	51	5 1131	5 1031	5.1139	6.24515	1 268	5.8106	5 793	5 834	6 24515	1 268

In view of all these data, it is evident how unwarranted are their comments on the nonphysical nature of our results and the (contrary to observation!) "degeneracy" in symmetric and antisymmetric modes. They, also, obtain the same results. Compare, for instance, their values k_{cn1} $(n = 3, 4, 5)$ for symmetric and antisymmetric TE modes. They are almost identical, exhibiting the same "degeneracy" as ours. In our case, degeneracy, if it exists, is due to the limitation $k_c d \ll 1$ and is restricted to order $(k_c d)^2$. It is obvious from our theory that even and odd k_c would differ if terms of order $(k_c d)^4$ could be maintained. The results of Vishen *et al.*, being so close to ours, actually verify this conclusion.

Finally, in Table II (TM modes) in Vishen *et al.*, the results for the antisymmetric modes should be moved down one line to restore the correct correspondence with the results of the symmetric modes on the left.

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[1]A. Vishen, G. P. Srivastava, G. S. Singh, and F. Gardiol, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 292–294, Feb. 1986.

Additional Comments[2]

M. DAVIDOVITZ[3]

Although Vishen *et al.* refer to the early work of Veselov and Semenov [6], no comparison is made between their formulation and the one found in [6]. I fail to recognize any essential differences between the two approaches. Furthermore, the authors state that the technique "based on Graf's addition theorem for Bessel functions" was developed by Singh and Kohtari [2], whereas in fact it appears that the same technique was employed in [6] considerably earlier.

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Reply[4] *by A. Vishen, G. S. Singh, and F. E. Gardiol*[5]

We would like to thank Dr. Fikioris, Dr. Roumeliotis, and Dr. Davidovitz for their interest and comments on our paper. We welcome this opportunity to clarify some aspects and hope in this way to clear up some confusion.

The cutoff wavenumbers, which are characterized by $\beta = 0$, have been considered in our paper [1], as is indicated by its title. This implies that in this work $k = k_c$, so that the error which Fikioris and Roumeliotis claim to have located just does not exist.

It seems that the basic spirit behind the approach discussed in [2] and used in our paper has been overlooked, resulting in comments on our statement "a rigorous mathematical derivation". The use of the addition theorems for the Bessel functions in the context of a solution to the Helmholtz equation is certainly not new; the same had been used some 40 years ago while studying eccentric control-rod problems in thermal nuclear reactors [3]. The Fourier expansions which we have made use of do provide a more general technique for solving problems with complicated boundaries. In fact, Nagaya and coworkers [4, and references therein] have already applied this approach in studying the vibration of membranes and plates of various geometries. However, for the geometry under discussion, our approach leads to the infinite set of linear equations identical with those given in [5] and [6].

One finds from the tables in [5] that symmetric and antisymmetric modes of a given higher order have the same value of g_{nm} up to the eighth decimal place. This gives a degeneracy which is simply an artifact of an approximate but algebraically tedious approach. The authors of [5] have not developed a form which would support their claim for the onset of bifurcation by inclusion of the fourth-order terms. On the other hand, they state that "the agreement for $k_c d < 1$ (sometimes even $k_c d > 1$) is remarkable." In contrast, we find that when $g_{nm} \simeq -6$ [5, tables II and III], the cutoff wavenumber goes to zero for $k_c d \simeq 0.4$ and even becomes negative for $k_c d \gtrsim 0.4$. This is clearly meaningless, referred to in [1] as "nonphysical."

One may be able to circumvent the bizarre situation if one considers the work in [5] to have a mode-dependent range of validity. But then this work loses its significance. One may even question the wisdom of using an approximate method whose validity for each mode requires that it be checked by comparison with experimental or exact theoretical work like ours. Of course, the numerical resolution in our work requires repetitive calculations. But this is hardly relevant nowadays when ample computer power is available. Furthermore, one does not have to worry over the validity of results for any particular value of $k_c d$.

It was noted in [1] that some of the cutoff values obtained lie outside (but near) the bounds reported by Kuttler [7]. Also, some of the values obtained by the approximate methods could not (as yet) be obtained with the rigorous approach. These two kinds of discrepancies are still unresolved; the comments by Fikioris and Roumeliotis do not provide any new insight on these issues.

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Comments on "Millimetric Nonreciprocal Coupled-Slot Finline Components"

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Abstract — **A statement is made concerning the feasibility of nonreciprocal components with ferrites magnetized in the direction of propagation.**

In the paper in question,[1] an isolator is presented operating with a ferrite magnetized in the direction of propagation inside waveguide with constant cross section.

Since only reciprocal devices with these properties have become known until now (Reggia–Spencer phase shifter), some general remarks should be made concerning the features of devices with gyrotropic media.

Let us consider an arbitrary waveguide section W (Fig. 1) between the planes A_1 and A_2, which are in the xy plane of a Cartesian coordinate system. The waveguide is completely or partly filled with gyromagnetic material with the permeability tensor

$$\vec{\mu} = \mu_0 \begin{pmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (1)$$

i.e., the magnetic bias field is applied in $+z$ direction. At A_1, we assume an arbitrary transversal field distribution $\vec{E}_1(x,y)$, $\vec{H}_1(x,y)$. This yields a field distribution $\vec{E}_2(x,y), \vec{H}_2(x,y)$ at A_2.

Inserting a magnetic wall M in the xy plane and applying image theory yields the waveguide section W' with the field

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[1]L. E. Davis and D B. Sillars, *IEEE Trans Microwave Theory Tech.*, vol MTT-34, pp 804–808, July 1986.